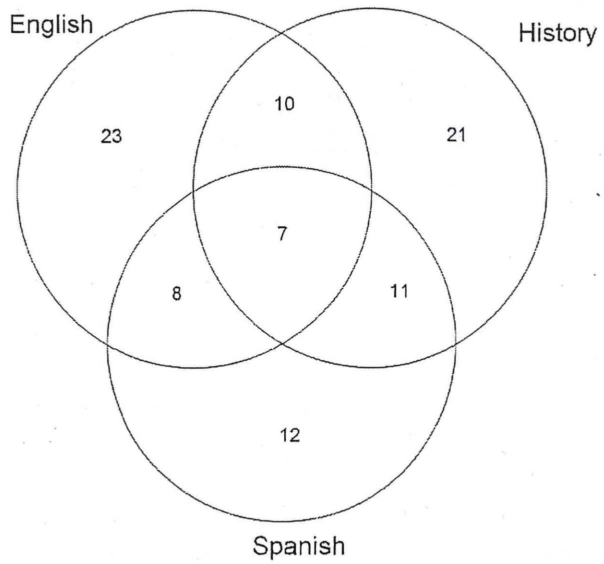
10.3 ~ Mutually Exclusive Events and Venn Diagrams



Objectives:

1. *Explore mutually exclusive events.*
2. *Use Venn diagrams as a tool for breaking down compound events into mutually exclusive events.*
3. *Understand the addition rule for finding the probabilities of events described by a Venn diagram.*
4. *Differentiate between mutually exclusive events and independent events.*

Probabilities for a Venn diagram follow two basic rules. First, there is a probability indicated for each separate region into which the enclosing rectangle is divided. Second, the probabilities for all of these (non-overlapping) regions must add up to one.

A. How many students take History? $10+7+21+11=49$

B. How many students take Spanish and English? $8+7=15$

C. How many students take Spanish or English? $12+11+7+8+10+23=71$

D. How many students take only English? 23

E. How many students take Spanish, English, and History? 7

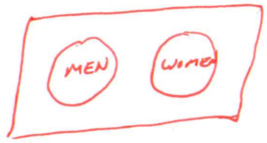
F. How many students take Spanish, English, or History? $23+10+8+7+21+11+12=92$

G. How many do not take English? $92-23=69$

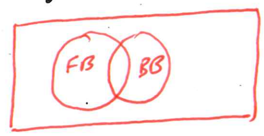
H. What is the probability a randomly selected person takes Spanish?

$$\frac{8+7+11+12}{92} = \frac{38}{92} = \frac{19}{46} = .413 = 41.3\%$$

Give an example of a mutually exclusive event.



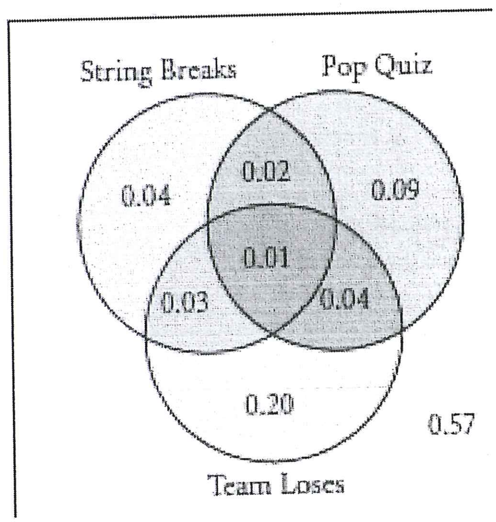
Now give an example of an event that is not mutually exclusive.



Example A: Melissa has been keeping record of probabilities of events involving:

I. Her orchestra string breaking during orchestra rehearsal (Event B).
II. A pop quiz in math (Event Q).
III. Her team losing in gym class (Event L).

Although the three events are not mutually exclusive, they can be broken into eight mutually exclusive events. These events and their probabilities are shown in the Venn diagram below.



a. What is the meaning of the region labeled 0.01?

String breaks, pop quiz, and team loses.

b. What is the meaning of the region labeled 0.03?

String breaks, team loses, no pop quiz

c. What is the probability of either a pop quiz or Melissa's team losing today? .02 + .09 + .01 + .04 + .03 + .2

.39

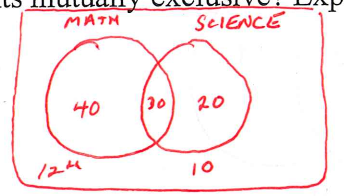
d. Find the probability of a pretty good day, $P(\text{not } B \text{ and not } Q \text{ and not } L)$. This means not string breaks and no quiz and no loss.

.57

Investigation – Addition Rule

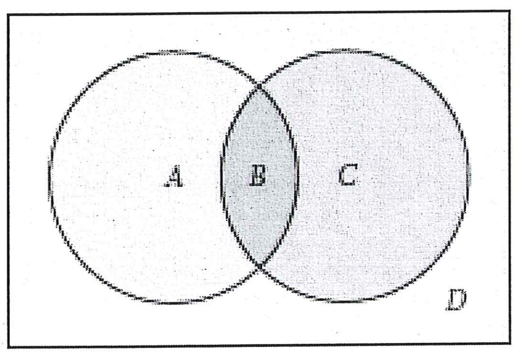
Of the 100 students in 12th grade, 70 are enrolled in mathematics, 50 are in science, 30 are in both subjects, and 10 are in neither subject.

Step 1 "A student takes mathematics" and "a student takes science" are two events. Are these events mutually exclusive? Explain.



NOT MUTUALLY EXCLUSIVE – STUDENTS CAN BE IN BOTH MATH & SCIENCE

Step 2: Complete a Venn diagram similar to the one below that shows enrollments in mathematics and science courses:



Step 3: Use the number of students in your Venn diagram to calculate probabilities:

1. $P(\text{JUST MATH}) = \frac{40}{100} = .4$ $\qquad P(\text{MATH}) = \frac{70}{100} = .7$
2. $P(\text{JUST SCIENCE}) = \frac{20}{100} = .2$ $\qquad P(\text{SCIENCE}) = .5$
3. $P(\text{MATH \& SCIENCE}) = \frac{30}{100} = .3$
4. $P(\text{MATH OR SCIENCE}) = \frac{90}{100} = .9$

Step 4: Explain why the probability that a randomly chosen student takes mathematics or science _$P(M$ or $S)$, does not equal $P(M) + P(S)$.

$P(m) + P(s) \neq P(m \text{ or } S)$

$.7 + .5 \neq .9$

THOSE THAT TOOK BOTH WERE COUNTED TWICE

Step 5: Create a formula for calculating $P(M$ or $S)$ that includes the expressions $P(M)$, $P(S)$, and $P(M$ ~~or~~ $S)$.
AND

$P(m \text{ or } S) = P(m) + P(s) - P(m \text{ and } s)$

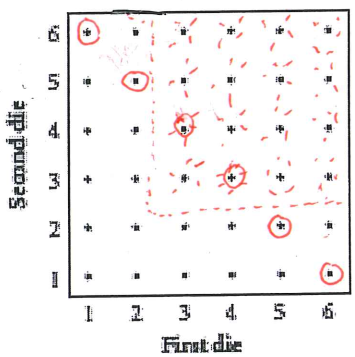
$= .7 + .5 - .3$

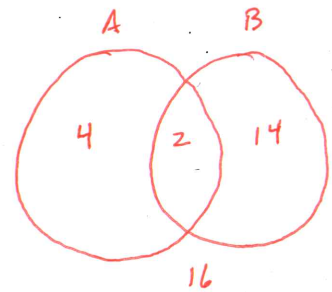
$= 1.2 - .3$

$= .9$

Step 6: Suppose two dice are tossed. Draw a Venn Diagram to represent the events:

A = "sum is 7"
B = "both dice > 2"





Find the probabilities in parts a – e:

a. $P(A)$ $\frac{6}{36} = \frac{1}{6} = .1\overline{6}$

b. $P(B)$ $\frac{16}{36} = \frac{4}{9} = .\overline{4}$

c. $P(A \text{ and } B)$ $\frac{2}{36} = \frac{1}{18} = .0\overline{5}$

d. P(A or B) $\frac{20}{36} = \frac{5}{9} = .\overline{5}$

e. $P(\text{not A and not B})$ $\frac{16}{36} = \frac{4}{9} = .\overline{4}$

f. Find $P(A \text{ or } B)$ by using a rule or formula similar to your response in step 5.
$P(A) + P(B) - P(A \text{ and } B) = \frac{6}{36} + \frac{16}{36} - \frac{2}{36} = \frac{20}{36} = \frac{5}{9} = .\overline{5}$

Step 7: Complete the statement: For any two events A and B,

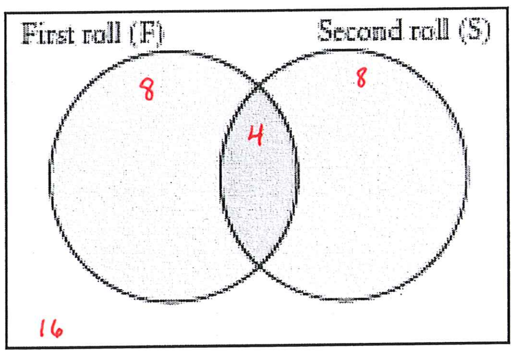
$P(A \text{ or } B) =$ $P(A) + P(B) - P(A \text{ and } B)$

The General Addition Rule

If n_1 and n_2 represent event 1 and event 2, then the probability that at least one of the events will occur can be found by adding the probabilities of the events and subtracting the probability that both will occur.

$$P(n_1 \text{ or } n_2) = P(n_1) + P(n_2) - P(n_1 \text{ and } n_2)$$

Example 2: The probability that a rolled die comes up 3 or 6 is $\frac{1}{3}$. What's the probability that a die will come up 3 or 6 on the first *or* second roll.



$P(1) + P(2) - P(1 \text{ and } 2)$

$\frac{1}{3} + \frac{1}{3} - \frac{1}{3} \cdot \frac{1}{3}$

$\frac{2}{3} - \frac{1}{9}$

$\frac{6}{9} - \frac{1}{9}$

$\frac{5}{9} = .\overline{5}$

How can we tell if two events are mutually exclusive?

The probability of the two events will equal one if they make up all possible outcomes.

$P(A \text{ and } B) = 0$

How can we tell if two events are independent?

$P(B/A) = P(B)$

or

$P(A/B) = P(A)$